

Combinations

The combination method of counting is used only when **order is not important** and **repetition is not allowed**. For example, when we are choosing members for a committee we are counting the number of combinations.

The formula for the number of combinations of n things taken r at a time follows:

$${}_n C_r = \frac{n!}{r!(n-r)!}$$

Example 1

You go to a local store to buy some movies for your family. You have a choice of six different categories: cartoon, comedy, documentary, musical, mystery, and western. You can only afford to buy four. How many different movie selections can you make if you cannot choose more than one from the same category?

Order is not important since the four movies you buy will be the same regardless of the order you selected them. Think of them as a group.

Repetition is not allowed since you cannot select two movies from the same category.

Solution - the Combination Formula – 6 categories taken 4 at a time where **order is not important** and **repetition is not allowed**.

You can make fifteen different movie selections.

$${}_6 C_4 = \frac{6!}{4!(6-4)!} = \frac{6!}{4!2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = \frac{30}{2} = 15$$

Example 2

How many ways can you make a sundae with two different toppings if five toppings are available?

Order is not important since the two toppings you put on the sundae will be the same regardless of the order you selected them.

Repetition is not allowed because the toppings must be different.

You can top the sundae ten different ways.

$${}_5 C_2 = \frac{5!}{2!(5-2)!} = \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = \frac{20}{2} = 10$$

Example 3

You are trying to figure out what courses to take next semester. You need to take a math class, an English class, and two humanities classes. How many different selections are possible if you can choose from 7 math classes, 3 English classes, and 5 humanities classes? Assume no conflict between class times.

Order is not important since you are just selecting which courses to take and there are no conflicting class times.

Repetition is not allowed because you will not be enrolled in more than one section of any course.

$$\begin{aligned} {}_7C_1 \cdot {}_3C_1 \cdot {}_5C_2 &= \frac{7!}{1!(7-1)!} \cdot \frac{3!}{1!(3-1)!} \cdot \frac{5!}{2!(5-2)!} \\ &= 7 \cdot 3 \cdot 10 \\ &= 210 \end{aligned}$$

You can select your courses two-hundred ten different ways.

Summary of the three counting methods

	Order important	Repetition Allowed
Fundamental Counting Principle	Yes	Yes or no
Permutation	Yes	No
Combinations	No	No

Notice that when repetition is allowed, you must use the Fundamental Counting Principle.